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Team Optimization Formulation

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Game Theory and Wireless Networks

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Outline

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Goal

The goal is to provide a perspective on game theory in wireless networking, present an overview of current approaches, and discuss game theory's future potential in networking research.

Why Game Theory?

- The microprocessor revolution has created distributed systems with significant processing power in individual nodes → independent decision makers.
- These system are connected to each with a variety wired/wireless communication technologies resulting in networked systems → interaction between decision makers.
- The systems share various resources (but often have only local information) → competition for available resources (resource allocation).

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Why Game Theory?

Recent developments in networking show that it is becoming more distributed, user-centric, and social.

- **More user-centric and flexible networking**: cognitive and software-defined radio
- **More distributed and novel networks**: femtocells, TV white space use (super Wi-Fi)
- **More social**: more powerful and ubiquitous individual devices blur the boundaries between network elements and its users, e.g. ad-hoc, delay tolerant, sensor networks

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Advantages of a Game Theoretic Approach

Advantages of the game theoretic approach include

- comprehensive mathematical modeling capabilities
- enabling
  - optimal and robust allocation of network resources
  - automatic decision making for security
- development of a decision and control framework for making resource allocation decisions in a principled manner.

Game Theory Basics

- GT studies multi-person decision making, and analyzes how decision-makers interact.
- We focus here exclusively on strategic (noncooperative) games.
- A strategic game consists of
  - players, who are decision makers acting selfishly,
  - actions chosen from a strategy (action) space, which is the set of all actions available to player(s),
  - outcomes (payoffs), which quantize gain or loss of players,
  - information structure (flow), characterizing how much each player knows about other's actions.

Types of Games

- 2-person versus N-person *(players)*
- finite versus infinite, discrete versus continuous kernel *(actions)*
- pure strategy versus mixed strategy *(actions)*
- normal form (matrix) versus extensive form *(actions)*
- zero (constant) sum versus nonzero sum *(payoffs)*
- static versus dynamic *(information)*

Network Games

- Autonomous parts of the networked systems (such as mobiles or other devices generating data traffic) are modeled as players.
- Players interact and compete with each other on the same system for limited and shared resources: e.g. quality of service, bandwidth...
- Players are associated with cost functions, which they minimize by choosing a strategy from a well defined strategy space.
- Nash equilibrium (NE) provides an appropriate solution concept, which is (approximately) optimal w.r.t. a global objective function.

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Interference Management

- Interference management aims to achieve optimal allocation of the electromagnetic spectrum, which is a limited resource (information theory)
- This is a primary resource allocation problem in wireless networks (others: battery, processing, greenIT)
- Various uplink and downlink transmission characteristics can be manipulated to achieve this objective, e.g. transmission power levels, frequency multiplexing (OFDM), medium access control (MAC), antenna properties (MIMO/beamforming)
- The goal can be expressed in terms of user Signal-to-interference ratio (SIR) levels

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Interference Management: Example

Uplink power control as a simple example game-theoretic formulation:

- The objective of (uplink) interference control is to regulate the transmission power level of each mobile in order to obtain and maintain a satisfactory quality of service or SIR level.
- In many types of wireless networks such as CDMA, signals of the users interfere and affect each other's service (SIR) level.
- In data networks, unlike in voice communication, SIR requirements vary from one user to another.
- A mobile has no access to information on other player's power level or preferences. Therefore, use of strategic game theory is appropriate.

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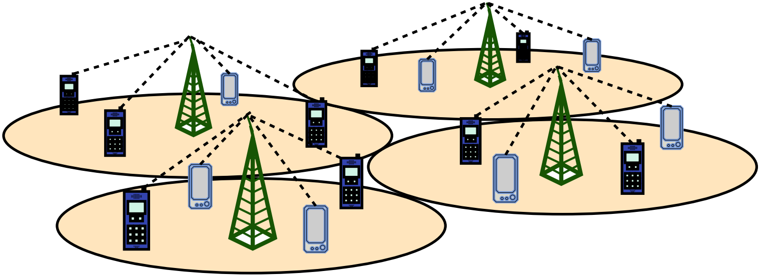
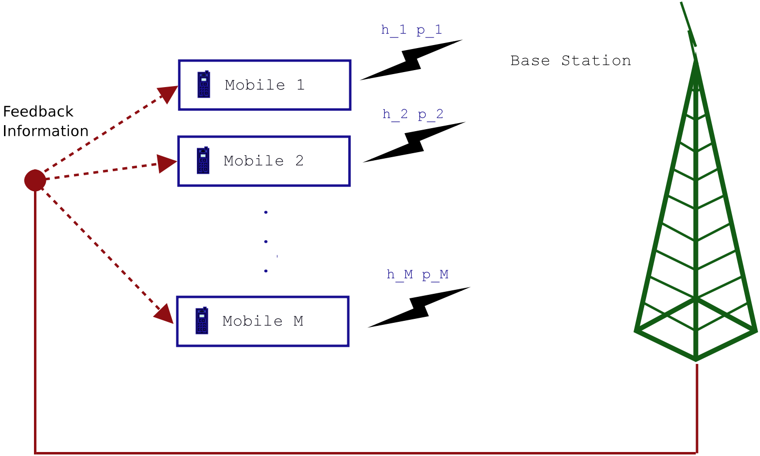
A Multicell Wireless Network



Distributed Interference Control



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A Network Model

- The system consists of $\mathcal{L} := \{1, \ldots, \bar{L}\}$ cells, with M_l users in cell l.
- Define $0 < h_{ij} < 1$ as the channel gain. Let secondary interference effects from neighboring cells be modeled as background noise, of variance σ^2.
- The i^{th} mobile transmits with an uplink power level of $p_i \leq p_{i,max}$, which is received at the BS j as $x_{ij} := h_{ij} p_i$. Then, SIR obtained by mobile i is

$$\gamma_{ij} := \frac{L h_{ij} p_i}{\sum_{k \neq i} h_{kj} p_k + \sigma^2}$$

A Cost Function

- Each mobile is associated with a cost function:

$$J_i(x_i, \mathbf{x}_{-i}, h_i) = P_i(x_i) - U_i(\gamma_i(\mathbf{x}))$$

- The benefit (utility) function, $U_i(\gamma_i)$ quantifies the user demand for quality of service or SIR level.
- The "pricing" function, $P_i(x_i)$ is imposed to limit the interference, and hence, improve the system performance. It can also be interpreted as a **cost on the battery** usage.

The Nash Equilibrium (NE)

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Definition

The Nash equilibrium is defined as a set of strategies (and corresponding set of costs), with the property that no player can benefit by modifying its own strategy while the other players keep theirs fixed.

If $\mathbf{x}$ is the strategy vector of players and X is the strategy space such that $\mathbf{x} \in X \ \ \forall \mathbf{x}$, then $\mathbf{x}^*$ is in NE when x_i^* of any i^{th} player satisfies

$$\min_{x_i} J_i(x_i, \mathbf{x}^*_{-i}),$$

where J_i is the cost function of the i^{th} player and $\mathbf{x}^*_{-i}$ is the equilibrium strategies of all other players.

System Dynamics and Stability

- Each mobile uses a gradient algorithm to solve its own optimization problem. The update scheme of the i^{th} mobile is:

$$\dot{p}_i = \frac{dp_i}{dt} = -\lambda_i \frac{\partial J_i}{\partial p_i}$$

- In terms of the received power level, x_i, at the BS:

$$\dot{x}_i = \frac{dU_i}{d\gamma_i} \frac{L \lambda_i h_i^2}{\sum_{j \neq i} x_j + \sigma^2} - \lambda_i h_i \frac{dP_i}{dx_i} := \phi_i(\mathbf{x}).$$

- Define the quadratic and radially unbounded Lyapunov function

$$V := \sum_{l \in \mathcal{L}} V_l = \sum_{l \in \mathcal{L}} \sum_{i \in \mathcal{M}_l} \phi_i^2(\mathbf{x})$$

System Dynamics and Stability

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Theorem

*Assume that there exists a unique NE in a static multicell CDMA wireless network, where each mobile stays connected to the respective BS for all times. Then, the system is globally asymptotically stable **if***

$$L > m_l\,(M_{eff} - 1),\ \forall l \in \mathcal{L}$$

where M_{eff} is defined as the largest cluster of users who have a nonnegligible effect on each other's SIR levels.

Remark 1. A large class of logarithmic utility functions, $U_i = u_i \log(k\gamma_i + 1)$, where $k > 1$ and $u_i > 0$ are scalar parameters, satisfy the sufficient assumptions for system stability.

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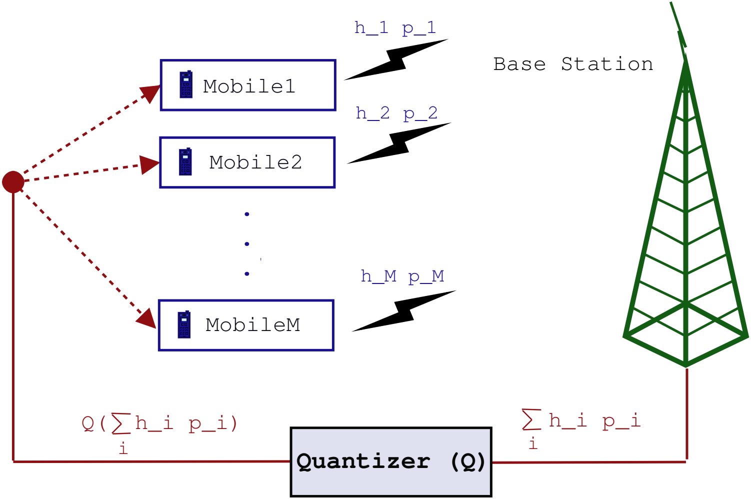
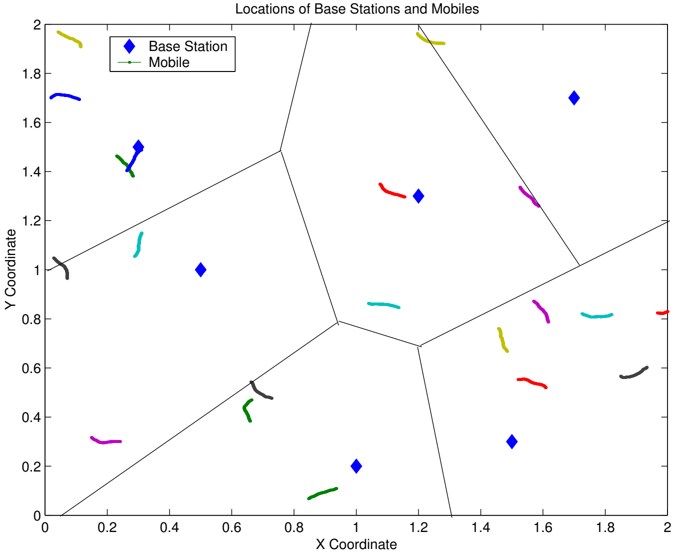
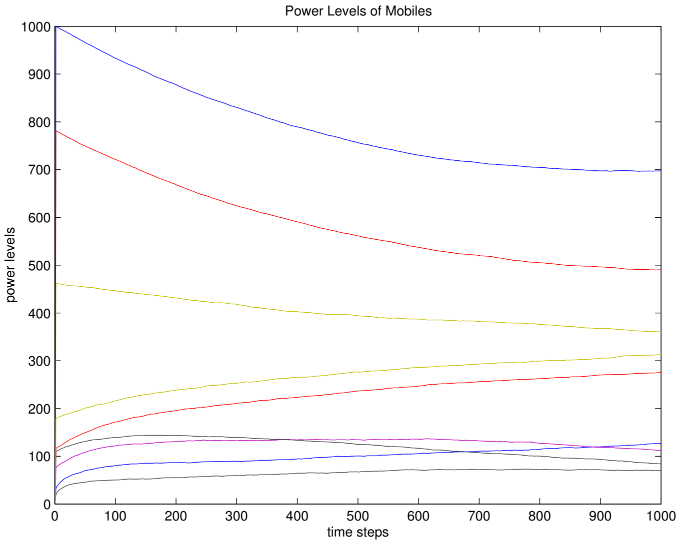
Communication Constraints





Locations of base stations and the paths of mobiles.



Power levels of selected mobiles with respect to time.

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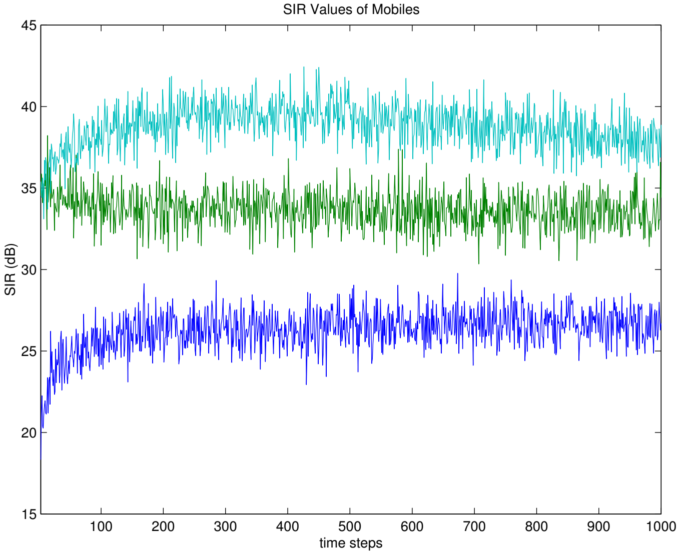

SIR values of selected mobiles (in dB) versus time.

Power Control as Team Optimization

- Power control in multicell CDMA wireless networks can also be studied as a team optimization problem.
- Each mobile attains its individual fixed target SIR level by transmitting with minimum possible power level.
- Using a Lagrangian relaxation approach[1], two decentralized dynamic power control algorithms: primal and dual power update are developed.
- Their global stability is established utilizing Lyapunov theory.

(1) F. Kelly, A. Maulloo, and D. Tan, "Rate control in communication networks: shadow prices, proportional fairness and stability," *JORS*, 1998.

Problem Formulation

- Each mobile i has a fixed target SIR level, $\bar{\gamma}_i$.
- Let $C_i(p_i)$ be the i^{th} mobile's cost function on its transmission power (battery usage and willingness to achieve higher SIR levels). We assume $C_i(p_i)$ to be strictly convex and continuously differentiable.
- Then, the system problem is

$$\min_{\mathbf{p}} \sum_{i=1}^{M} C_i(p_i) \text{ such that } \gamma_i \geq \bar{\gamma}_i,\ 0 \leq p_i \leq p_{max}\ \forall i \in \mathcal{M}.$$

-

$$\text{Alternatively:} \quad \min_{\mathbf{p}} \sum_{i=1}^{M} C_i(p_i) \text{such that}$$
$$\mathbf{p} \in \Omega := \{\mathbf{p} \in \mathbb{R}^M : \mathcal{A}\mathbf{p} \geq \beta, \text{ and}$$
$$0 \leq p_i \leq p_{max}\ \forall i \in \mathcal{M}\}.$$

User and Network Problems

- Using Lagrangian decomposition, the system problem is divided into a User$_i$ problem for the i^{th} user and a Network one.
- Define $r_i := p_i q_i \; \forall i$, where $q_i = \mathrm{row_i}(\mathcal{A}^{\mathrm{T}})\lambda$.
- As in the case of congestion control;

$$\text{User}_i : \min_{r_i} C_i\left(\frac{r_i}{q_i}\right) - r_i, \;\; r_i \geq 0$$

$$\text{Network} : \; \min_{\mathbf{p}} \sum_{i \in \mathcal{M}} -r_i \log(p_i), \;\; \mathbf{p} \in \Omega.$$

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Primal Update Algorithm

$$\dot{p}_i = \frac{dp_i}{dt} = \kappa_i \left(-\frac{\partial C_i(p_i)}{\partial p_i} + q_i \right), \; i = 1, 2, \ldots, M$$

$$\mathbf{q}(\mathbf{p}) = \mathcal{A}^T \rho(\mathbf{p}),$$

Theorem

The primal power update algorithm admits a unique equilibrium, $\mathbf{p}^*$, that solves a relaxed version of the system problem. Furthermore, it is globally asymptotically stable.

Dual Update Algorithm

We now study the associated dual problem, given by

$$\max_{\mu \geq 0} \sum_{i \in \mathcal{M}} C_i \left((C_i')^{-1}(q_i(\mu)) \right) - q_i(\mu) \cdot (C_i')^{-1}(q_i(\mu)) + \mu^T \beta,$$

where $q_i := \mathrm{row_i}(\mathcal{A}^{\mathrm{T}})\mu$. Then, the dual power update algorithm is:

$$\dot{\mu}_i = \kappa_i(b_i - \mathrm{row_i}(\mathcal{A})\mathbf{p}(\mu)), \;\; \mathrm{q_i}(\mu) := \mathrm{row_i}(\mathcal{A}^{\mathrm{T}})\mu,$$

$$p_i = (C_i')^{-1}(q_i(\mu)), \; i = 1, 2, \ldots, M \; .$$

Theorem

The dual power update algorithm admits a unique equilibrium that solves the dual system problem. Furthermore, it is globally asymptotically stable.

Game Design

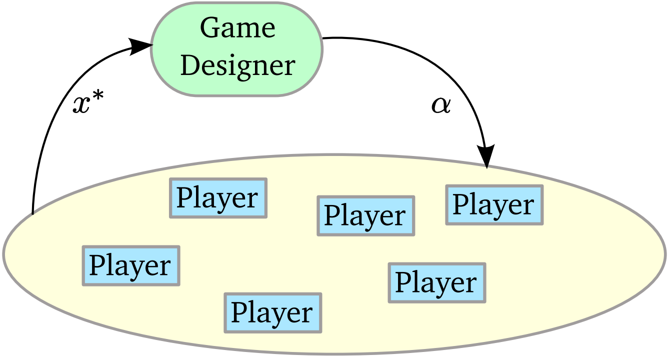

Figure: The rules or pricing mechanisms within a game can be set by a "designer" to influence the outcome.

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Game Classes

Consider **a specific class of games**, $\mathcal{G}1$, by assuming a cost structure of the form

$$J_i(\alpha_i, \mathbf{x}) = \alpha_i p_i(\mathbf{x}) - U_i(\mathbf{x}), \quad (1)$$

where the functions p_i and U_i are smooth and chosen in such a way that there exists at least a single NE in the game.

Further define **another class of games**, $\mathcal{G}2$, as a special case of $\mathcal{G}1$ with additional conditions on the cost structure, such that they admit a unique NE solution.

Static Game Design

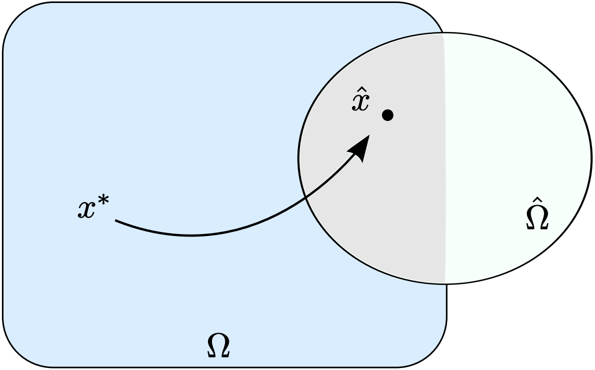

Figure: Game design involves controlling the game dynamics such that the NE, $\mathbf{x}^* \in \Omega$ is moved to a feasible desired region $\hat{\Omega} \cap \Omega$ or a specific optimal point $\hat{\mathbf{x}}$.

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Static Optimization

Theorem

For games of class $\mathcal{G}2$ with the cost structure given in (1) and under complete information assumption, affine pricing of the form, $\alpha\, p(\cdot)$, is sufficient to locate the unique NE point of the game to any desirable feasible point, $\hat{\mathbf{x}} \in \Omega$, as long as

$$\frac{\partial p_i(\hat{\mathbf{x}})}{\partial x_i} \neq 0, \quad \forall i.$$

Dynamic Control of Games

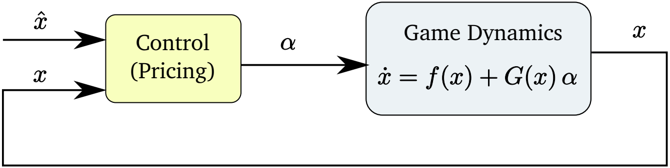

Figure: Feedback control of the game (NE, $\mathbf{x}^*$) using pricing α as the control parameter and $\hat{\mathbf{x}}$ as the desired reference signal.

Dynamic Control of Games

The counterpart of the feasibility question in static game optimization relates in the dynamic control setting to the controllability of the system.

For games of class $\mathcal{G}2$, the game dynamics are:

$$\dot{x}_i = -\frac{\partial J_i(\mathbf{x})}{\partial x_i} = \frac{\partial U_i(\mathbf{x})}{\partial x_i} - \frac{\partial p_i(\mathbf{x})}{\partial x_i}\,\alpha_i \;\; \forall i, \tag{2}$$

where α acts as the feedback control on the outcome of the game. Alternatively, in vector form:

$$\dot{\mathbf{x}} = f(\mathbf{x}) + \sum_{i=1}^{N} g_i(\mathbf{x})\,\alpha_i = f(\mathbf{x}) + G(\mathbf{x})\,\alpha \tag{3}$$

Design under Information Constraints

Define a strictly concave and smooth social welfare function

$$\mathcal{U}(\mathbf{x}) := \sum_i U_i(\mathbf{x})$$

that admits a global maximum

$$\hat{\mathbf{x}} = \arg\max_{\mathbf{x}} \sum_i U_i(\mathbf{x}).$$

The player cost function is $J_i(\alpha_i, \mathbf{x}) = \alpha_i p_i(\mathbf{x}) - U_i(x_i)$.

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Social Welfare Maximization

The social maximum is defined easily via the first order optimality conditions

$$\frac{\partial \mathcal{U}}{\partial \mathbf{x}}(\hat{\mathbf{x}}) = 0,$$

where

$$\frac{\partial \mathcal{U}}{\partial \mathbf{x}}(\mathbf{x}) = \left[\begin{array}{ccc} \sum_j \frac{\partial U_j}{\partial x_1}(\mathbf{x}) & \ldots & \sum_j \frac{\partial U_j}{\partial x_N}(\mathbf{x}) \end{array} \right].$$

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Pricing Dynamics

The social maximum is shown to coincide with the unique equilibrium (and NE) of the following pricing mechanism

$$\dot{\alpha}_i = \sum_j \sum_k \frac{\partial U_j}{\partial x_k^*} \frac{\partial x_k^*}{\partial \alpha_i} \quad \forall i \tag{4}$$

If these pricing dynamics are on a slower time scale than the game dynamics, then the system designer can obtain sufficiently accurate estimates of $\partial U_i(\mathbf{x}^*)/\partial x_i$ and $\partial x_i^*/\partial \alpha_i$.

Pricing Dynamics

Theorem

Define an objective function $\mathcal{U}(\mathbf{x}) := \sum_i U_i(\mathbf{x})$ *which admits a unique inner global maximum* $\hat{\mathbf{x}} = \arg\max_{\mathbf{x}} \mathcal{U}(\mathbf{x})$ *under suitable assumptions for user utilities* $U_i \ \forall i$ *in a class* $\mathcal{G}2$ *game. Then, the pricing mechanism (4) ensures that the NE point of the underlying game,* $\mathbf{x}^*$*, globally asymptotically converges to the maximum of the social welfare function,* $\hat{\mathbf{x}}$*, if the Jacobian matrix of the mapping* $\mathcal{T}$ *with respect to pricing vector* α*, defined as*

$$H(\alpha) := \frac{\partial \mathbf{x}^*}{\partial \alpha}(\alpha) = \left[\frac{\partial x_i^*}{\partial \alpha_j}(\alpha) \right], \quad i, j = 1, \ldots, N,$$

is non-singular.

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Example

Let,

$$J_i(\alpha_i, \mathbf{x}) = \alpha_i \left(\sum_i x_i \right) - \beta_i \log(1 + s_i(\mathbf{x})),$$

where

$$s_i(\mathbf{x}) := \frac{h_i x_i}{\sum_{j \neq i} h_j x_j + \sigma}.$$

Then, under non-singularity conditions on the system matrix M, the unique NE is $\mathbf{x}^* = M^{-1}\,\mathbf{v}$, where

$$\mathbf{v} = [v_i], \quad v_i = \frac{\beta_i}{\alpha_i} - \sigma.$$

Here, x_i^* depends on all pricing parameters, α, and $H(\alpha)$ is not diagonal. However, one can still explicitly find $\partial x^* / \partial \alpha$, and show that under non-singularity conditions on M^T, H is non-singular.

Comparison

Three approaches:

- **Strategic games** - NE as solution concept
- **Team optimization** - Global objective achieved through distributed optimization
- **Game design** - manipulating the game to move NE to a global optimum

Strategic game + game design $\equiv$ Team optimization

Open issues: cheating and collusion in game design

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Incentives and Cheating

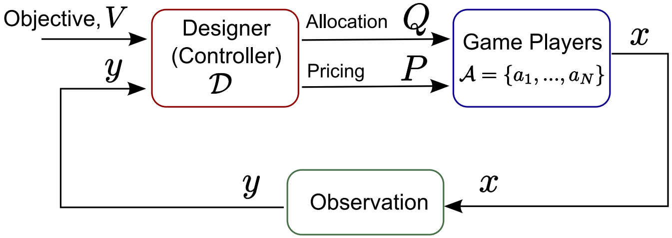

Figure: Auction mechanisms

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Incentives and Cheating

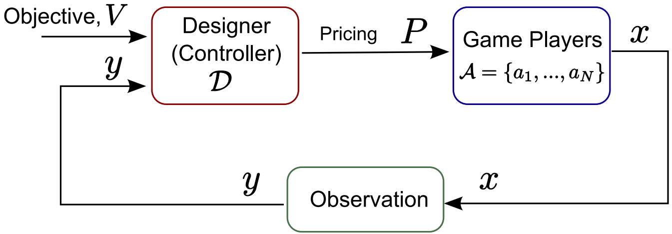

Figure: Pricing mechanisms

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Decision Making in Security

Table: Security Decisions in a Company

Decision-Maker	Time Scale	Actions
CEO CIO	Years Months	App store or open platform policies and major security investments
Dept. head	Days-hours	Dept. rules, allocation of manpower, budget
IDPS	Seconds	Block ports, access control packet inspection

How to make these decisions in a principled way using on analytical models and methods?

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Security Games

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Table: Components of a Security Game

Component	Description
Players	attacker and defender
Action Space	set of attacks or defensive measures
Outcome	the cost and benefit to players
Information structures	players observe each others' actions or not

Security Games in Wireless Networks

- Selfish nodes aim to transmit to a common receiver and maximize their throughput
- Malicious nodes try to disrupt communication by simultaneous transmissions and jamming.
- A jamming game on a slotted random access channel.

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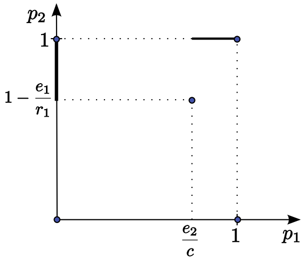
Security Games in Wireless Networks

The cost functions of the attacker, $\mathcal{P}^A$ or Player 2, and defender, $\mathcal{P}^D$, or Player 1:

$$J_1(p) := p_1 e_1 - r p_1 (1 - p_2),$$

$$J_2(p) := p_2 e_2 - c p_2 p_1 .$$

This game again admits multiple Nash Eq. solutions.



Application Areas: Security Games

- Intrusion detection and optimal response
- Jamming games in wireless networks
- Vehicular networks
- Interconnected systems
- Revocation games
- Social networks - community trust game
- Security investment games
- Cooperative games for security risk management
- Location privacy games

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GameSec Conference

GameSec 2010, the inaugural Conference on Decision and Game Theory for Security will take place in Deutsche Telekom Laboratories, on the campus of Technical University Berlin, Germany, on November 22-23, 2010. More information at *http://www.gamesec-conf.org/*

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Future Directions

This is not even half of the story! Open directions include

- Network games with MIMO, AOFDM, femtocells
- Decision making under limited information
- Learning in games (Q-learning, Fictitious play)
- Adversarial mechanism design
- Security games
- Ad-hoc, delay-tolerant, social networks
- Privacy and trust issues (e.g. location privacy, digital trust)

Research Challenges and Questions

How can we

- bring new learning schemes to game theory?
- further focus on networking effects and handle complexity?
- incorporate robustness further into game theoretic models?
- develop mechanisms resistant to adversarial behavior?
- model social aspects in problems such as privacy and trust?
- capture rare events in security game models?

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Thank you and Questions?

Further information is available at:
http://www.tansu.alpcan.org

I would like to thank PerAda for sponsoring this plenary talk!